
CANDIDATE NUMBER

SYDNEY GRAMMAR SCHOOL

2017 Trial Examination

FORM VI

MATHEMATICS 2 UNIT

Thursday 10th August 2017

General Instructions

- Reading time — 5 minutes
- Writing time — 3 hours
- Write using black pen.
- Board-approved calculators and templates may be used.

Total — 100 Marks

- All questions may be attempted.

Section I – 10 Marks

- Questions 1–10 are of equal value.
- Record your answers to the multiple choice on the sheet provided.

Section II – 90 Marks

- Questions 11–16 are of equal value.
- All necessary working should be shown.
- Start each question in a new booklet.

Collection

- Write your candidate number on each answer booklet and on your multiple choice answer sheet.
- Hand in the booklets in a single well-ordered pile.
- Hand in a booklet for each question in Section II, even if it has not been attempted.
- If you use a second booklet for a question, place it inside the first.
- Write your candidate number on this question paper and hand it in with your answers.
- Place everything inside the answer booklet for Question Eleven.

Checklist

- SGS booklets — 6 per boy
- Multiple choice answer sheet
- Reference sheet
- Candidature — 91 boys

Examiner

RCF

SECTION I - Multiple Choice

Answers for this section should be recorded on the separate answer sheet handed out with this examination paper.

QUESTION ONE

Which of the following is equal to $\dfrac{1}{\sqrt{5}-\sqrt{2}}$?

(A) $\dfrac{\sqrt{5}+\sqrt{2}}{3}$

(B) $\dfrac{-\sqrt{5}-\sqrt{2}}{3}$

(C) $\dfrac{\sqrt{5}-\sqrt{2}}{3}$

(D) $\dfrac{\sqrt{2}-\sqrt{5}}{3}$

QUESTION TWO

Which expression is equal to $\displaystyle\int e^{3x}\,dx$?

(A) $e^{3x}+C$

(B) $3e^{3x}+C$

(C) $\dfrac{e^{3x}}{3}+C$

(D) $\dfrac{e^{3x+1}}{3x+1}+C$

QUESTION THREE

What is the domain of the function $f(x)=\dfrac{1}{\sqrt{x+1}}$?

(A) $x<-1$

(B) $x\le -1$

(C) $x>-1$

(D) $x\ge -1$

Examination continues next page ...

QUESTION FOUR

What is the solution of $3^x = 5$?

(A) $x = \dfrac{\ln 5}{3}$

(B) $x = \dfrac{\ln 5}{\ln 3}$

(C) $x = \dfrac{5}{\ln 3}$

(D) $x = \ln\left(\dfrac{5}{3}\right)$

QUESTION FIVE

A parabola has directrix $x = 4$ and focus at the origin. What is the equation of the parabola?

(A) $y^2 = 8(x - 2)$

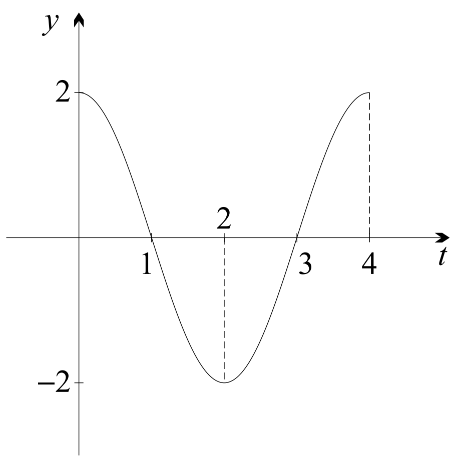
(B) $y^2 = -8(x + 2)$

(C) $y^2 = -8(x - 2)$

(D) $y^2 = 8(x + 2)$

QUESTION SIX

(a)



The diagram above shows one period of the function $y = 2\cos bt$. What is the value of the constant b?

(A) $b = \frac{\pi}{2}$

(B) $b = \pi$

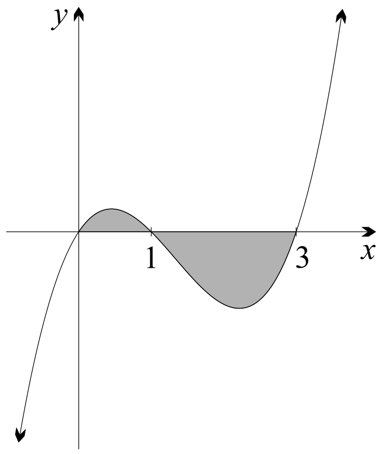
(C) $b = 2\pi$

(D) $b = 4\pi$

Examination continues overleaf ...

QUESTION SEVEN

(a)



The diagram shows the cubic $y = x(x-1)(x-3)$ with x-intercepts at $(1, 0)$, $(3, 0)$ and the origin. Which expression gives the correct value for the total shaded area?

(A) $\displaystyle\int_0^3 x(x-1)(x-3)\,dx$

(B) $\displaystyle\left|\int_0^3 x(x-1)(x-3)\,dx\right|$

(C) $\displaystyle\left|\int_0^1 x(x-1)(x-3)\,dx\right| + \int_1^3 x(x-1)(x-3)\,dx$

(D) $\displaystyle\int_0^1 x(x-1)(x-3)\,dx - \int_1^3 x(x-1)(x-3)\,dx$

QUESTION EIGHT

Using the trapezoidal rule with three function values gives which expression as an approximation for $\displaystyle\int_2^4 x\ln x\,dx$?

(A) $\frac{1}{2}(2\ln 2 + 3\ln 3 + 4\ln 4)$

(B) $5\ln 2 + 3\ln 3$

(C) $\frac{1}{2}(2\ln 2 + 4\ln 3 + 4\ln 4)$

(D) $\ln 2 + 3\ln 3 + 4\ln 4$

QUESTION NINE

Which expression is a term of the geometric series $-2y + 6y^2 - 18y^3 + \cdots$?

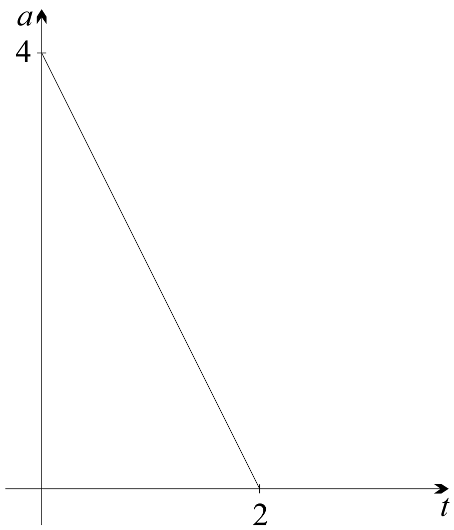
(A) $39366y^9$

(B) $-39366y^9$

(C) $39366y^{10}$

(D) $-39366y^{10}$

QUESTION TEN



A particle is moving along the x-axis. The graph shows its acceleration a m/s^2 at time t seconds. Initially the particle has a velocity of -2 m/s. What is its velocity after 2 seconds?

(A) 0 m/s

(B) 2 m/s

(C) 4 m/s

(D) 8 m/s

End of Section I

SECTION II - Written Response

Answers for this section should be recorded in the booklets provided.

Show all necessary working.

Start a new booklet for each question.

QUESTION ELEVEN (15 marks) Use a separate writing booklet. **Marks**

(a) State the exact value of $\tan \frac{2\pi}{3}$. **1**

(b) Solve $2^x = \dfrac{1}{8}$. **1**

(c) Evaluate $\displaystyle\sum_{k=1}^{3} k^3$. **1**

(d) Write $3\log_2 8$ in its simplest form. **1**

(e) Solve $|x - 5| = 2$. **2**

(f) Solve $\cos\theta = -\dfrac{1}{2}$ for $0 \leq \theta \leq 2\pi$. **2**

(g) (i) Simplify $1 - \cos^2\theta$. **1**

(ii) Hence, prove the identity **1**

$$\cot\theta(1 - \cos^2\theta) = \sin\theta\cos\theta.$$

(h) Differentiate with respect to x:

(i) $y = 3 + \dfrac{2}{x}$ **1**

(ii) $y = \ln(3x - 2)$ **1**

(iii) $y = \cos 3x$ **1**

(i) Find the hundredth term in the arithmetic sequence **2**

$$1 + \sqrt{2}, 1 + 3\sqrt{2}, 1 + 5\sqrt{2}, 1 + 7\sqrt{2}, \ldots$$

Examination continues next page ...

QUESTION TWELVE (15 marks) Use a separate writing booklet. **Marks**

(a) Find:

(i) $\int e^{\frac{x}{2}}\,dx$ **1**

(ii) $\int \frac{4}{x}\,dx$ **1**

(iii) $\int \frac{5}{x^2}\,dx$ **1**

(b) The equation $2x^2 - 3x - 1 = 0$ has roots α and β. Without solving the equation, find the value of:

(i) $\alpha + \beta$ **1**

(ii) $\alpha\beta$ **1**

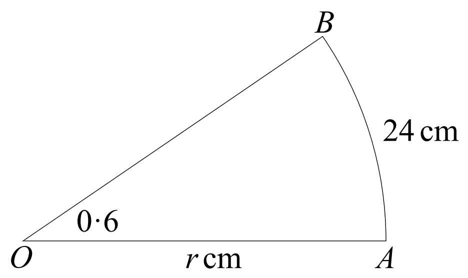
(iii) $\alpha^2 + \beta^2$ **2**

(c) Differentiate the following, giving your answers in fully factorised form:

(i) $y = x^2 e^{3x}$ **2**

(ii) $y = \dfrac{x^2}{\sin x}$ **2**

(d)



The diagram shows a sector of a circle with radius r cm. The $\angle AOB = 0{\cdot}6$ radians and the arc AB is 24 cm.

(i) Find the value of r. **1**

(ii) Calculate the area of the sector AOB. **1**

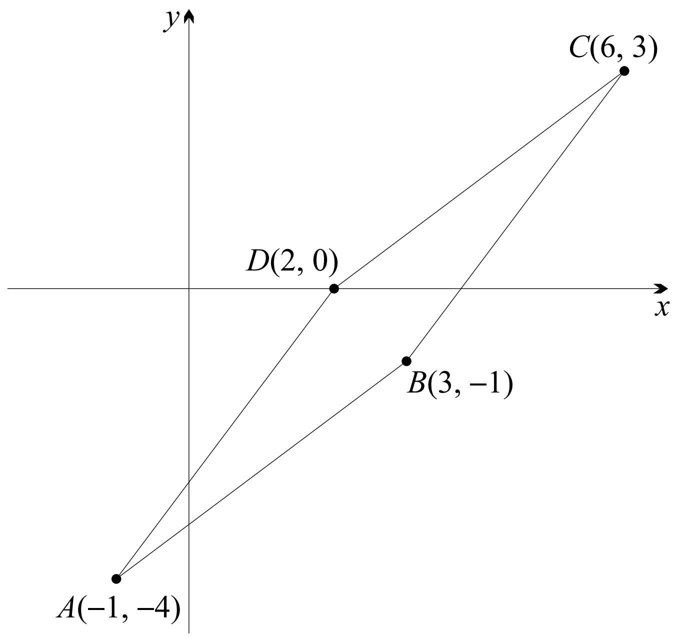
(e) Evaluate $\displaystyle\int_0^{\frac{\pi}{6}} \cos 2x\,dx$. **2**

Examination continues overleaf ...

QUESTION THIRTEEN (15 marks) Use a separate writing booklet. **Marks**

(a) Find the equation of the tangent to the curve $y = \sin \pi x$ at the point where $x = \frac{1}{3}$. **3**

(b)



The points $A(-1,-4), B(3,-1), C(6,3)$ and $D(2,0)$ form a parallelogram as shown in the diagram.

(i) Show that the equation of the line through A and B is $3x - 4y - 13 = 0$. **2**

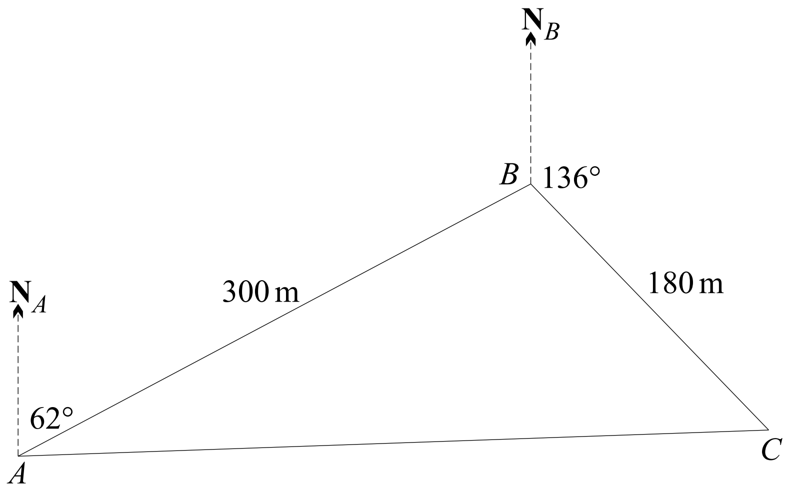
(ii) Find the perpendicular distance from D to the side AB. **1**

(iii) Hence, find the area of the parallelogram. **1**

(iv) By considering the gradients of the diagonals of the parallelogram show that $ABCD$ is actually a rhombus. **1**

(c) Solve the equation $x^6 + 7x^3 - 8 = 0$. **2**

(d)



Dave flies a drone 300 metres from A to B on a bearing of 062°. It then changes direction to fly 180 m from B to C on a bearing of 136°.

(i) Explain why $\angle ABC = 106^o$, giving clear geometric reasons. **1**

(ii) Use the cosine rule to find AC, the distance the drone is now from its starting point, correct to the nearest metre. **1**

(iii) Find $\angle BCA$, correct to the nearest minute. **1**

(iv) Hence, or otherwise, find the bearing for the third stage of its flight from C directly back to A. Give your answer as a true bearing accurate to the nearest degree. **2**

Examination continues overleaf ...

QUESTION FOURTEEN (15 marks) Use a separate writing booklet. **Marks**

(a) Nigel is in his third year of working for Reliable Real Estate. During this time his annual salary has increased by the same percentage every year. This year his salary is \$52 920, which is an increase of \$2 520 on the previous year.

(i) Find the percentage increase that is applied to his salary each year. **1**

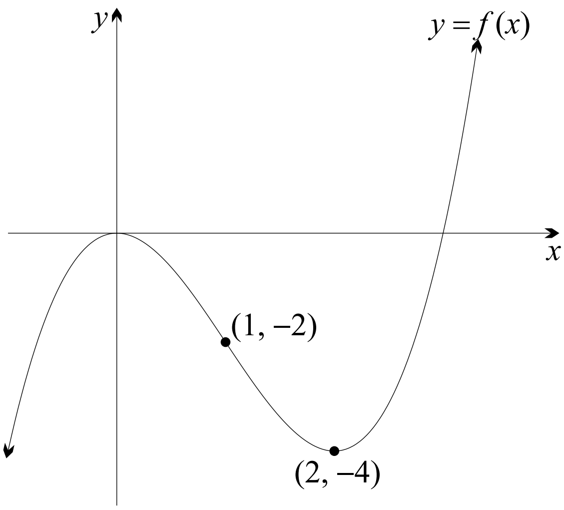
(ii) Show that his starting salary was \$48 000. **1**

(iii) If he stays with the company, what will his annual salary have become by his tenth year of employment? Give your answer to the nearest dollar. **1**

(iv) How much will Nigel have been paid by the company during these ten years of employment? Give your answer to the nearest dollar. **1**

(v) How many full years must he work for the company to have earned over a million dollars in total? **2**

(b)



The function $y = f(x)$ is graphed in the diagram above. The points $(2, -4)$ and the origin are stationary points and the point $(1, -2)$ is a point of inflexion.

(i) State the values of x for which $f'(x) = 0$. **1**

(ii) For what values of x is $f''(x) < 0$? **1**

(iii) Sketch the gradient function $y = f'(x)$, given that the gradient of the tangent at the point of inflexion is -3. Mark on your sketch the co-ordinates of all intercepts with the axes and any stationary points. **2**

(c) Consider the quadratic equation $x^2 - kx + (3 - k) = 0$.

(i) Find the discriminant in factorised form. **1**

(ii) For what values of k does the equation have real roots? **2**

(iii) If the sum of the roots is twice the product of the roots, find the value of k. **2**

Examination continues next page ...

QUESTION FIFTEEN (15 marks) Use a separate writing booklet. **Marks**

(a) The velocity of a particle moving along the x-axis is given by $\dot{x} = 10t - 40$ where x is the displacement from the origin in metres at time t seconds. Initially the particle is 100 metres to the right of the origin.

(i) Show that the acceleration of the particle is constant. **1**

(ii) Find the time when the particle is at rest. **1**

(iii) Show that the position of the particle after five seconds is 25 metres to the right of the origin. **2**

(iv) Find the total distance travelled in the first five seconds. **2**

(b) The age of a wooden bowl is to be found using carbon dating. The amount C of radioactive Carbon-14 will be measured and compared to the amount originally present C_0 when the tree was cut down for use as timber. The amount of Carbon-14 measured t years after a tree is felled is given by

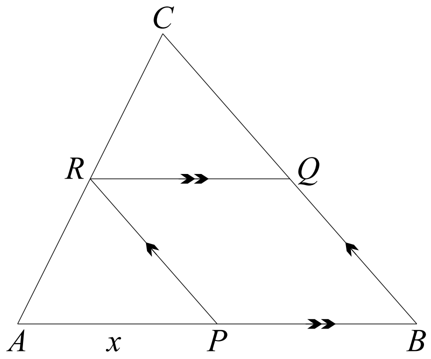
$$C = C_0\, e^{-kt}.$$

(i) Given that only half the Carbon-14 remains after 5 750 years, show that $k = 1{\cdot}21 \times 10^{-4}$, correct to three significant figures. **1**

(ii) Archaeologists suspect that the bowl is 20 000 years old. If this is true, what percentage of the original amount of Carbon-14 should be found when the bowl is tested? Give your answer to the nearest integer percentage. **1**

(iii) When the test results come back, it is discovered that 15% of the original amount of Carbon-14 remains. How old does this suggest the bowl actually is? Give your answer to the nearest hundred years. **2**

(c)



In $\triangle ABC$, a point P is chosen on the side AB such that the length AP is x. The line through P, parallel to side BC, meets AC at point R. The line through R, parallel to AB, meets BC at Q, as shown in the diagram above.

(i) Show that $\triangle APR$ is similar to $\triangle RQC$. **2**

(ii) Explain why **2**

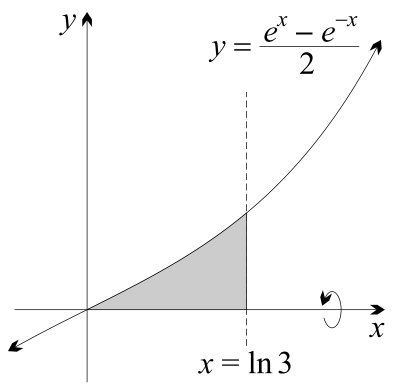
$$\frac{AR}{RC} = \frac{x}{c-x}$$

where the length AB is c.

(iii) Hence find a simplified expression for the ratio of the areas of $\triangle APR$ to $\triangle RQC$. **1**

QUESTION SIXTEEN (15 marks) Use a separate writing booklet. **Marks**

(a) **4**



The diagram shows the region bounded by the curve $y = \dfrac{e^x - e^{-x}}{2}$, the co-ordinate axes and the vertical line $x = \ln 3$. This region is rotated about the x-axis to form a solid of revolution with volume V. Find the volume, writing your answer in the form

$$V = \frac{\pi}{2}\left[\frac{a}{b} - \ln 3\right]$$

where a and b are integers.

(b) Geoff has found an idyllic beach house and needs to borrow \$800 000 from the bank to finance his purchase. The loan and interest is to be repaid in equal monthly instalments of $\$M$, at the end of each month, over a 25 year period. The reducible interest will be charged at 4·8% p.a. and compounded monthly. As an extra inducement the bank agrees that the first six months of the loan can be interest free, although Geoff will begin making repayments from the end of the first month.

Let $\$A_n$ be the amount owing to the bank at the end of n months.

(i) Given $A_6 = 800\ 000 - 6M$, write down an expression for A_7 and show that the amount owing after eight months is given by **2**

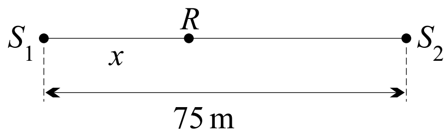
$$A_8 = (800\ 000 - 6M)1{\cdot}004^2 - M(1{\cdot}004 + 1).$$

(ii) Hence show **1**

$$A_n = (800\ 000 - 6M)1{\cdot}004^{n-6} - 250M(1{\cdot}004^{n-6} - 1).$$

(iii) Calculate the monthly instalment, $\$M$, Geoff will need to pay in order to repay the loan on time. Give your answer correct to the nearest cent. **2**

(c) At a music concert two speaker towers are placed 75 metres apart. The intensity of sound produced by a speaker tower of power P at a distance x metres from the tower is given by $I = \dfrac{4P}{\pi x^2}$.



The speakers in tower S_1 have a power output of P but the older speakers in tower S_2 produce 25% less power. Rebecca R stands in between the two towers and x metres from tower S_1 as shown in the diagram above.

(i) Show that the sound intensity produced by speaker tower S_2 at point R is $I = \dfrac{3P}{\pi(75 - x)^2}$. **1**

Given that the total sound intensity from both speaker towers I_T at point R is

$$I_T = \frac{4P}{\pi x^2} + \frac{3P}{\pi(75 - x)^2},$$

(ii) find $\dfrac{dI_T}{dx}$. **2**

(iii) Hence find the value of x which minimises the sound intensity so that Rebecca knows where best to stand to enjoy the concert. Give your answer to three significant figures. **3**

End of Section II

END OF EXAMINATION

RCF

MATHEMATICS (2U) TRIAL 2017

① $\dfrac{1}{\sqrt{5}-\sqrt{2}} \times \dfrac{(\sqrt{5}+\sqrt{2})}{(\sqrt{5}+\sqrt{2})} = \dfrac{\sqrt{5}+\sqrt{2}}{5-2}$ (A)

$= \dfrac{\sqrt{5}+\sqrt{2}}{3}$

② $\int e^{3x}\,dx = \frac{1}{3}e^{3x} + C$ (C)

③ $\dfrac{1}{\sqrt{x+1}}$ $\sqrt{x+1} > 0$ (C)

$x+1 > 0$

$x > -1$

④ $3^x = 5$

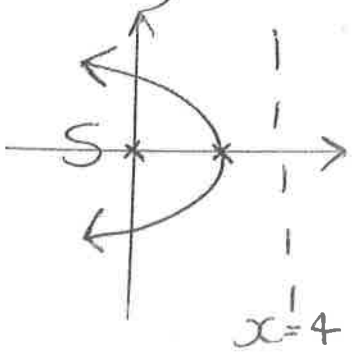
$(\log_e)$ $(\log_e)$

$\ln 3^x = \ln 5$

$x \ln 3 = \ln 5$ (B)

$x = \dfrac{\ln 5}{\ln 3}$

⑤ $S(0,0)$ $x=4$ $V(2,0)$ (C)



$y^2 = -8(x-2)$

⑥ $T = \dfrac{2\pi}{b}$ $4 = \dfrac{2\pi}{b}$ $b = \dfrac{2\pi}{4}$ (A)

$= \pi/2$

⑦ (D) since area between $x=1$ & $x=3$ below axis

$$\left|\int_1^3 f(x)\,dx\right| = -\int_1^3 f(x)\,dx$$

⑧ $\int_2^4 x\ln x\,dx$

x	2	3	4
$x\ln x$	$2\ln 2$	$3\ln 3$	$4\ln 4$

$$\doteqdot \tfrac{1}{2}(2\ln 2+3\ln 3)+\tfrac{1}{2}(3\ln 3+4\ln 4)$$
$$= \tfrac{1}{2}(2\ln 2+6\ln 3+4\ln 4)$$
$$= \tfrac{1}{2}(2\ln 2+6\ln 3+8\ln 2)$$
$$= 5\ln 2+3\ln 3$$

(since $\ln 4=\ln 2^2 = 2\ln 2$)

(B)

⑨ $-2y+6y^2-18y^3+\ldots$

GP $a=(-2y)$
$r=(-3y)$

$t_9 = ar^8 = (-2y)\times(-3y)^8$
$= -2\times 3^8 y^9 = -13122y^9$

$t_{10} = ar^9 = -3y\times t_9$
$= +39366y^{10}$

(C)

⑩ $a=4-2t$

$V=\int a\,dt = 4t-t^2+C$

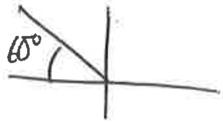
$t=0$ $V=(-2)$ $\therefore C=-2$

$V=4t-t^2-2$

$V(2) = 8-4-2 = 2\,m/s$

(B)

11 a) $\tan\frac{2\pi}{3} = \underline{(-\sqrt{3})}$



b) $2^x = \frac{1}{8}$

$= 2^{-3}$

$\underline{x = -3}$

c) $\sum_{k=1}^{3} k^3 = 1^3 + 2^3 + 3^3$

$= 1 + 8 + 27$

$\underline{= 36.}$

d) $3\log_2 8 = 3\log_2 2^3$

$= 9\log_2 2$

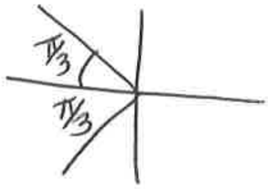
$\underline{= 9}$

e) $|x-5| = 2$

$x - 5 = 2$ or $x - 5 = (-2)$

$x = 7$ or $x = 3$

f) $\cos\theta = -\frac{1}{2}$



$\underline{\theta = \frac{2\pi}{3}, \frac{4\pi}{3}}$

g) (i) $1 - \cos^2\theta = \underline{\sin^2\theta}$

(ii) $LHS = \cot\theta\,(1 - \cos^2\theta)$

$= \frac{\cos\theta}{\sin\theta} \times \sin^2\theta$

$= \cos\theta\sin\theta = RHS$

h) (i) $y = 3 + \frac{2}{x} = 3 + 2x^{-1}$

$\frac{dy}{dx} = 0 - 2x^{-2}$

$\underline{= -\frac{2}{x^2}}$

(ii) $y = \ln(3x-2)$

$\underline{\frac{dy}{dx} = \frac{3}{3x-2}}$

(iii) $y = \cos 3x$

$\underline{\frac{dy}{dx} = -3\sin 3x}$

i) $1+\sqrt{2}, 1+3\sqrt{2}, 1+5\sqrt{2}, \ldots$

AP, $a = 1+\sqrt{2}$ $d = 2\sqrt{2}$

$t_{100} = a + 99d$

$= 1 + \sqrt{2} + 198\sqrt{2}$

$\underline{= 1 + 199\sqrt{2}}$

12 a)(i) $\int e^{\frac{x}{2}}dx = \frac{e^{x/2}}{\frac{1}{2}} + C$

$= 2e^{x/2} + C$ ✓

(ii) $\int \frac{4}{x}dx = \int 4x^{-1}dx$

$= 4\ln x + C$ ✓

(iii) $\int \frac{5}{x^2}dx = \int 5x^{-2}dx$

$= \frac{5x^{-1}}{-1} + C$

$= \frac{-5}{x} + C$ ✓

b) $2x^2 - 3x - 1 = 0$

$a = 2 \quad b = (-3) \quad c = (-1)$

(i) $\alpha + \beta = -\frac{b}{a}$

$= \frac{3}{2}$ ✓

(ii) $\alpha\beta = \frac{c}{a}$

$= \left(-\frac{1}{2}\right)$ ✓

(iii) $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$ ✓

$= \left(\frac{3}{2}\right)^2 - 2\times\left(-\frac{1}{2}\right)$

$= \frac{9}{4} + 1$

$= \frac{13}{4}$ ✓ or $3\frac{1}{4}$

c)(i) $y = x^2 e^{3x}$

$u = x^2 \qquad v = e^{3x}$

$\frac{du}{dx} = 2x \qquad \frac{dv}{dx} = 3e^{3x}$

$\frac{dy}{dx} = 2xe^{3x} + x^2 3e^{3x}$ ✓

$= xe^{3x}[2 + 3x]$ ✓ Must be factored

(ii) $y = \frac{x^2}{\sin x}$

$u = x^2 \qquad v = \sin x$

$\frac{du}{dx} = 2x \qquad \frac{dv}{dx} = \cos x$

$\frac{dy}{dx} = \frac{2x\sin x - x^2\cos x}{\sin^2 x}$ ✓

$= \frac{x(2\sin x - x\cos x)}{\sin^2 x}$ ✓

d)(i) $l = r\theta$

$24 = r \times 0{\cdot}6$

$r = \frac{24}{0{\cdot}6}$

$= 40\text{cm}$ ✓

(ii) $A = \frac{1}{2}r^2\theta$

$= \frac{1}{2}\times 40^2 \times 0{\cdot}6$

$= 480\text{ cm}^2$ ✓

(e) $\int_0^{\pi/6} \cos 2x\,dx$

$= \left[\frac{1}{2}\sin 2x\right]_0^{\pi/6}$ ✓

$= \frac{1}{2}\sin\frac{\pi}{3} - \frac{1}{2}\sin 0$

$= \frac{\sqrt{3}}{4}$ ✓

13 a) $y = \sin \pi x \quad x = \frac{1}{3} \quad y = \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2} \quad \left(\frac{1}{3}, \frac{\sqrt{3}}{2}\right)$ ✓

$\frac{dy}{dx} = \pi \cos \pi x \quad \left(\frac{dy}{dx}\right)_{x=\frac{1}{3}} = \pi \cos\frac{\pi}{3} = \frac{\pi}{2}$ ✓

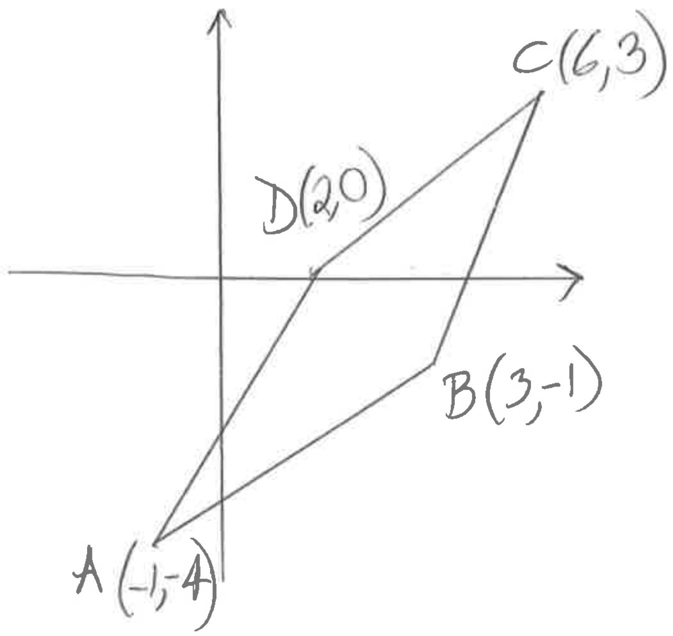
Eqn: $y - \frac{\sqrt{3}}{2} = \frac{\pi}{2}\left(x - \frac{1}{3}\right)$

$2y - \sqrt{3} = \pi x - \frac{\pi}{3}$

$6y - 3\sqrt{3} = 3\pi x - \pi$

$\underline{3\pi x - 6y + 3\sqrt{3} - \pi = 0}$ ✓

b)



(i) $m_{AB} = \frac{-1-(-4)}{3-(-1)} = \frac{3}{4}$ ✓

AB: $y-(-1) = \frac{3}{4}(x-3)$

$y+1 = \frac{3}{4}(x-3)$

$4y+4 = 3x-9$ SHOW ✓

$3x-4y-13=0$

(ii) $p = \frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}} \quad a=3 \quad b=(-4) \quad c=(-13)$

$x_1 = 2 \quad y_1 = 0$

$= \frac{|3\times 2+(-4)\times 0+(-13)|}{\sqrt{3^2+(-4)^2}}$

$= \frac{|-7|}{5}$

$= \frac{7}{5}u$ ✓

(iii) Area $= bh$

$= AB \times p$

$= 5 \times \frac{7}{5}$

$AB = \sqrt{4^2+3^2}$

$= \underline{7u^2}$ ✓

(iv) $m_{AC} = \frac{3-(-4)}{6-(-1)} = \frac{7}{7} = 1 \qquad m_{BD} = \frac{-1-0}{3-2} = -1$ ✓

$\therefore m_{AC} \times m_{BD} = -1$

$\therefore AC \perp BD \therefore$ Rhombus

c) $x^6 + 7x^3 - 8 = 0$

Let $u = x^3$

$u^2 + 7u^2 - 8 = 0$ ✓

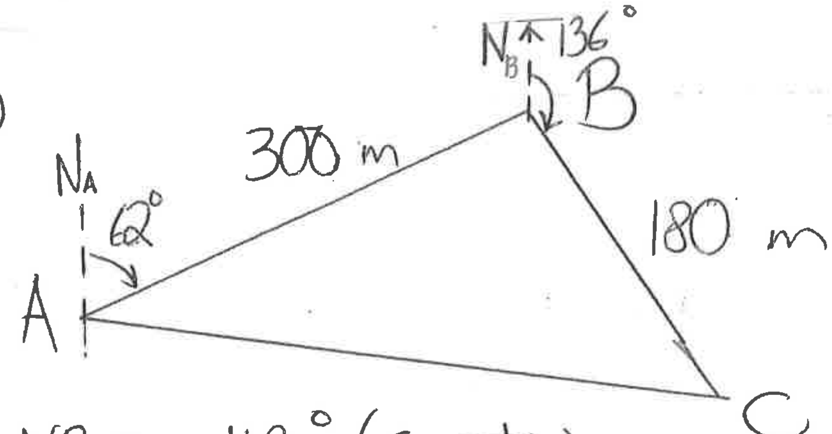
$(u+8)(u-1) = 0$

$u = 1$ or (-8)

$x^3 = 1$ or $x^3 = (-8)$

$\underline{x = 1}$ $\quad \underline{x = (-2)}$ ✓

d)



(i) $\angle NBC = 118°$ (Co-interior $\angle$ on // north lines)

$\therefore \angle ABC = 360 - 136 - 118$ (Angles at point B or Full Revolution)

$= 106°$

(ii) $AC^2 = 300^2 + 180^2 - 2 \times 300 \times 180 \times \cos 106°$

$= 90000 + 32400 - 108\,000 \cos 106°$

$= 152\,168.834\ldots$

$AC \doteqdot 390.088\ldots$ ✓

$\doteqdot \underline{390 \text{ metres}}$ ✓

(iii) Let $\angle BCA = \theta$

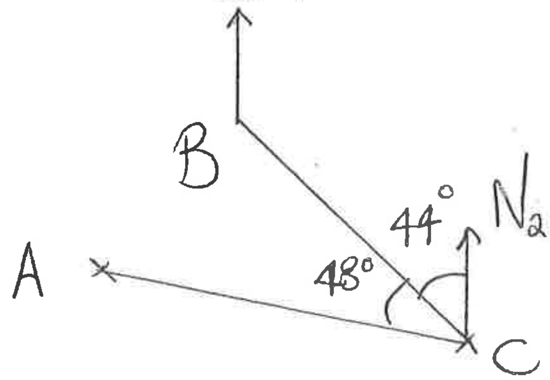
$$\frac{\sin\theta}{300} = \frac{\sin 106}{AC}$$

$$\sin\theta = \frac{300\sin 106}{AC}$$

$= 0.7392$ ✓

$\theta = 47°40'$ ✓ (NB: Obtuse value not possible)

(iv) Bearing from A to C is $360 - 44° - 47°40'$



since

$\angle BCN_2 = 44°$ (Cointerior on ∥ North lines) ✓

Bearing is 268° to nearest degree. ✓

14 a) Current \$52920 Interest \$2520
Previous \$50400

(i) % increase $\frac{2520}{50400} \times 100\% = \underline{5\%}$ ✓

(ii) Starting Salary. Decrease \$50400 (ie $\div 1.05$)

$$x \times 1.05 = 50400$$
$$x = \frac{50400}{1.05}$$
$$= \underline{\$48000}$$

✓ "Show"

(iii) In Seven Years t_{10}

GAP $a = 48000$ $r = 1.05$

$$t_{10} = ar^9 = 48000 \times 1.05^9$$
$$\doteqdot \underline{\$74\,464}$$ ✓

(iv) $S_{10} = \frac{a(r^n - 1)}{r - 1} = \frac{48000(1.05^{10} - 1)}{0.05}$ ✓

$$= \underline{\$603\,739}.$$

(v) $S_n > 1000\,000$

$$\frac{48000}{0.05}(1.05^n - 1) > 1000\,000$$
$$1.05^n - 1 > \frac{25}{24}$$
$$1.05^n > \frac{49}{24}$$ ✓
$$n \log 1.05 > \log \frac{49}{24}$$
$$n > \frac{\log \frac{49}{24}}{\log 1.05}$$

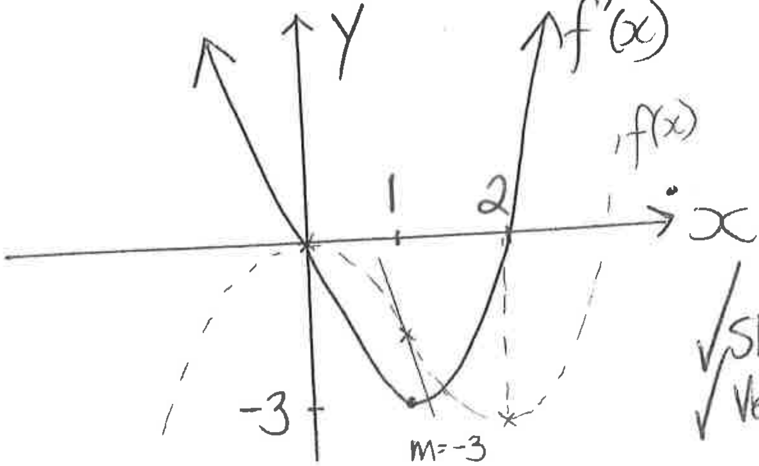
$n > 14.62$

ie Fifteen full years! ✓

b) (i) $f'(x)$ stat pts $\underline{x = 0 \text{ and } 2}$ ✓

(ii) $f''(x) < 0$ Concave down $\underline{x < 1}$ ✓

(iii)

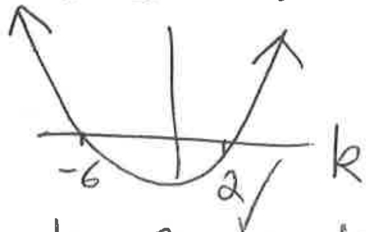


✓Shape
✓Vertex (1,-3)

c) $x^2 - kx + (3-k) = 0$ $\quad a = 1 \quad b = (-k)$
$c = (3-k)$

(i) $\Delta = (-k)^2 - 4 \times (3-k)$
$= k^2 + 4k - 12$
$= \underline{(k+6)(k-2)}$ ✓

(ii) Real Roots $\Delta \geq 0$
$(k+6)(k-2) \geq 0$



$\underline{k \geq 2}$ OR $\underline{k \leq (-6)}$ ✓

(iii) $\alpha + \beta = 2\alpha\beta$.
$-\frac{b}{a} = 2\frac{c}{a}$
$\therefore -b = 2c$ ✓
$k = 2(3-k)$
$k = 6 - 2k$
$3k = 6$
$\underline{k = 2}$ ✓

(15) a)(i) $\dot{x} = 10t - 40$

$a = \frac{d\dot{x}}{dt} = 10$ $\therefore$ Acceleration is constant at $10 m/s^2$ ✓ "show"

(ii) at rest $\dot{x} = 0$

$10t - 40 = 0$ ✓

$t = 4$ seconds

(iii) $x = \int \dot{x}\, dt$

$= 5t^2 - 40t + C$

$t = 0$ $x = 100$

$\therefore 100 = C$ ✓

$x = 5t^2 - 40t + 100$

$x(5) = 5 \times 25 - 40 \times 5 + 100$ ✓ "SHOW"

$= 125 - 200 + 100$

$= 25$

$\therefore$ 25 metres to the right

(iv) $0 \leq t < 4$ $\dot{x} < 0$ ie moving left

$x(4) = 5 \times 16 - 40 \times 4 + 100$

$= 80 - 160 + 100$

$= 20m$

distance travelled 80 metres ✓

$4 < t \leq 5$ $\dot{x} > 0$ ie moving right

distance 5 metres

Total distance 85 metres ✓

b) $C = C_0 e^{-kt}$

(i) $t = 5750 \quad C = \frac{C_0}{2}$

$\frac{1}{2} = e^{-k \times 5750}$

$2 = e^{5750k}$

$\ln 2 = 5750k$

$k = \frac{\ln 2}{5750}$ ✓SHOW

$\doteqdot 1.21 \times 10^{-4}$

(ii) $t = 20000$

$C = C_0 e^{-20000k}$

$= 0.0897\ldots C_0$

$\approx 9\%$ ✓

(iii) $C = 0.15C_0$.

$\therefore 0.15 = e^{-kt}$

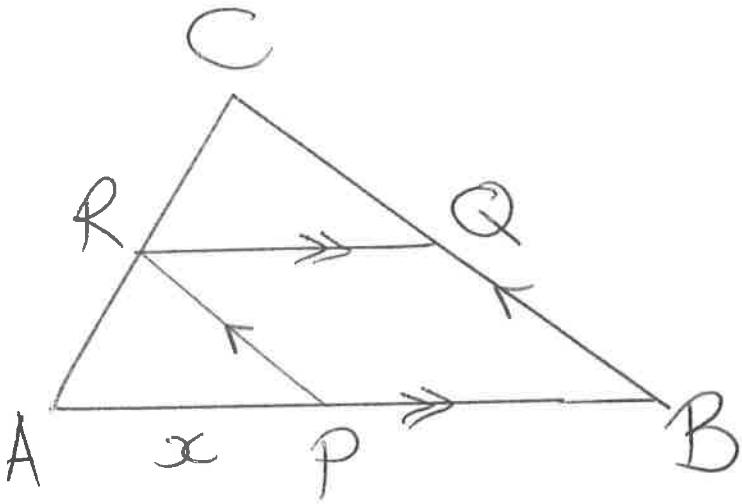
$\frac{20}{3} = e^{kt}$ ✓

$\frac{1}{k}\ln\left(\frac{20}{3}\right) = t$

$t = 15737.55\ldots$ ✓

≈ 15700 years old. ✓

c)



(i) in $\triangle APR$ & $\triangle RQC$

$\angle ARP = \angle RCQ$ (Corresponding $\angle$ $RP \parallel QC$) ✓ "SHOW"

$\angle PAR = \angle QRC$ (Corresponding $\angle$, $AP \parallel RQ$) ✓

$\therefore \triangle APR \parallel\!\!\parallel \triangle RQC$ (AA)

(ii) $\frac{AR}{RC} = \frac{AP}{RQ}$ (Corresponding sides in similar $\triangle$ in ratio) ✓ "EXPLAIN"

$AB = c \therefore PB = c - x \therefore RQ = c - x$ (Opposite sides of parallelogram PBQR)

$$\frac{AR}{RC} = \frac{x}{c-x}$$

(iii) $|APR| : |RQC|$

$\frac{1}{2} AP \times AR \times \sin \angle PAR : \frac{1}{2} RQ \times RC \times \sin \angle QRC$

since $\angle PAR = \angle QRC$

$AP \times AR : RQ \times RC$

$x \times \frac{x \times RC}{c-x} : RQ \times RC$

$\frac{x^2}{c-x} : c-x$ ✓

$\underline{x^2 : (c-x)^2}$

(or simply length scale factor $\frac{x}{c-x}$ ∴ area scale factor $\frac{x^2}{c-x^2}$ hence ratio $x^2 : (c-x)^2$)

(16)

a) $V = \pi \int_0^{\ln 3} \left(\frac{e^x - e^{-x}}{2} \right)^2 dx$ $\qquad V_x = \int \pi y^2 \, dx$

$= \frac{\pi}{4} \int_0^{\ln 3} e^{2x} - 2 + e^{-2x} \, dx$ ✓

$= \frac{\pi}{4} \left[\frac{e^{2x}}{2} - 2x - \frac{e^{-2x}}{2} \right]_0^{\ln 3}$ ✓

$= \frac{\pi}{4} \left[\left(\frac{e^{2\ln 3}}{2} - 2\ln 3 - \frac{e^{-2\ln 3}}{2} \right) - \left(\frac{e^0}{2} - 0 - \frac{e^0}{2} \right) \right]$

$= \frac{\pi}{4} \left(\frac{e^{\ln 9}}{2} - 2\ln 3 - \frac{e^{\ln 1/9}}{2} \right)$ ✓

$= \frac{\pi}{4} \left(\frac{9}{2} - 2\ln 3 - \frac{1}{18} \right)$

$= \frac{\pi}{4} \left(\frac{40}{9} - 2\ln 3 \right)$

$= \frac{\pi}{18} (20 - 9\ln 3)$

$= \frac{\pi}{2} \left(\frac{20}{9} - \ln 3 \right)$ ✓ $\quad a = 20 \quad b = 9$.

b) i) $A_6 = 800\,000 - 6M$

$4.8\%\text{pa} = \frac{4.8}{12}\%$ p.month $= 0.4\%$ per month

$A_7 = A_6 \times 1.004 - M$

$= \underline{800000 \times 1.004 - 6M \times 1.004 - M}$ ✓

$A_8 = A_7 \times 1.004 - M$

"Show"

$= 800000 \times 1.004^2 - 6M \times 1.004^2 - M \times 1.004 - M$ ✓

$= (800000 - 6M) \times 1.004^2 - M(1.004 + 1)$

(ii) Continuing pattern

$A_n = (800\,000 - 6M) \times 1.004^{n-6} - M(1.004^{n-7} + 1.004^{n-8} + 1.004^{n-9} + \ldots + 1)$

$$\left.\begin{aligned} &GP \quad a = 1 \quad r = 1.004 \quad (n-6) \text{ terms} \\ &S = \frac{1(1.004^{n-6} - 1)}{1.004 - 1} \\ &\quad = 250(1.004^{n-6} - 1) \end{aligned}\right\} \checkmark \text{Show}$$

$A_n = (800\,000 - 6M) \times 1.004^{n-6} - 250M(1.004^{n-6} - 1)$

(iii) 25 years = 300 months $\therefore n = 300 \quad A_{300} = 0$

$0 = (800\,000 - 6M) \times 1.004^{294} - 250M(1.004^{294} - 1)$ ✓

$6M \times 1.004^{294} + 250M(1.004^{294} - 1) = 800\,000 \times 1.004^{294}$

$M(256 \times 1.004^{294} - 250) = 800\,000 \times 1.004^{294}$

$M \doteqdot 4476.995\ldots$ ie $\underline{\$4477.00}$ ✓

c) $I = \dfrac{4P}{\pi x^2}$ $\quad 0 < x < 75$

$RS_2 = (75 - x)$ metres

$I_1 = \dfrac{4P}{\pi x^2}$ $\quad$ (i) $I_2 = \dfrac{75}{100} \times \dfrac{4P}{\pi(75-x)^2}$ ✓ "SHOW"

$$= \frac{3P}{\pi(75-x)^2}$$

(ii) $I_T = \dfrac{4P}{\pi x^2} + \dfrac{3P}{\pi(75-x)^2}$ ✓

$$\frac{dI_T}{dx} = \frac{-8P}{\pi x^3} + \frac{3P}{\pi} \; -2(75-x)^{-3}(-1)$$ ✓

$$= \frac{-8P}{\pi x^3} + \frac{6P}{\pi(75-x)^3}$$ ✓

$$= \frac{2P}{\pi}\left(\frac{3}{(75-x)^3} - \frac{4}{x^3}\right)$$

Stat Pt $\quad \dfrac{dI_T}{dx} = 0$

$$\frac{4}{x^3} = \frac{3}{(75-x)^3}$$ ✓

$$4(75-x)^3 = 3x^3$$

$$\left(\frac{75-x}{x}\right)^3 = \frac{3}{4}$$

$$\frac{75-x}{x} = \sqrt[3]{\frac{3}{4}}$$

$$\frac{75}{x} - 1 = \sqrt[3]{\frac{3}{4}}$$

$$\frac{75}{x} = 1 + \sqrt[3]{\frac{3}{4}}$$ ✓

$$x \doteq 39.2966\ldots$$ ✓

Nature

x	30	39.296	40
$\frac{dI_T}{dx}$	$-$	0	$+$

$x = 30$

$\frac{dI}{dx} = \frac{-4P}{30^3} + \frac{3P}{45^3}$

$-0.000115 \doteqdot = -\frac{7P}{60750} < 0$

$30^3 = 27000$
$35^3 = 42875$
$40^3 = 64000$
$45^3 = 91125$

$x = 40$

$\frac{dI}{dx} = \frac{-4P}{40^3} + \frac{3P}{35^3}$

$= \frac{41P}{5488000} = 0.00000747P$

> 0

$\therefore$ Minimum Turning pt ✓ Must show "min"

Only stat pt in domain hence global minimum

I_T is minimised when $x \doteqdot 39.3$ metres